

Exercise 3. (A. Hatcher, Algebraic Topology, p. 38, 7.) Define $f : S^1 \times I \rightarrow S^1 \times I$ by $f(\theta, s) = (\theta + 2\pi s, s)$, so f restricts to the identity on the two boundary circles of $S^1 \times I$. Show that f is homotopic to the identity by a homotopy f_t that is stationary on one of the boundary circles, but not by any homotopy f_t that is stationary on both boundary circles. [Consider what f does to the path $s \mapsto (\theta_0, s)$ for fixed $\theta_0 \in S^1$.]

Solution

We consider the function $f : S^1 \times I \rightarrow S^1 \times I$ given by

$$f(z, s) = (ze^{2\pi is}, s), \forall (z, s) \in S^1 \times I.$$

Obviously, the map $F : S^1 \times I \times I \rightarrow S^1 \times I$ given by

$$F(z, s, t) = (ze^{2\pi it}, s), \forall (z, s, t) \in S^1 \times I \times I,$$

is continuous. Moreover, we have

$$F(z, s, 0) = (z, s), F(z, s, 1) = (ze^{2\pi is}, s) = f(z, s), \forall (z, s) \in S^1 \times I$$

and

$$F(z, 0, t) = (z, 0), \forall (z, t) \in S^1 \times I.$$

Therefore, F is a homotopy from $id_{S^1 \times I}$ to f which is stationary on the circle $S^1 \times \{0\}$. In a similar manner we obtain the required result for the other boundary circle.

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